2641 Mathematics: Statistics 1

January 2004

Mark Scheme

1 (i)

Year	1	2	3	4	5	6	7
Points gained by team *A*	41	82	62	58	50	60	80
Points gained by team *B*	65	79	81	66	83	36	70
Ranks for team *A*	7	1	3	5	6	4	2
Ranks for team *B*	6	3	2	5	1	7	4
d	1	-2	1	0	5	-3	-2

B1 Correct ranks (or reverse)

M1 Attempt to find *d* (or d^2) from ranked data

$\Sigma d^2 = 1 + 4 + 1 + 0 + 25 + 9 + 4 = 44$, so

M1 Correct formula for Spearman *used*.

Spearman's Rank correlation coefficient $= 1 - \dfrac{6 \times 44}{7 \times 48}$

$= \mathbf{{}^{3}/_{14} = 0.214285\ldots}$

= 0.214 (3 s.f.)

A1 Correct answer (fraction or decimal) a.r.t. 0.214

4

(ii) Spearman's rank correlation coefficient shows that there is little agreement in the rankings gained by the two teams over the 7 seven years.

B1 Comment stating a "low" level of agreement

1

2 (i) $S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n} = 47\,520 - \frac{550 \times 803}{10}$ $= 3\,355$ $S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n} = 38\,500 - \frac{550^2}{10}$ $= 8\,250$ $S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n} = 65\,848.9 - \frac{803^2}{10}$ $= 1\,368$ $r = \frac{S_{xy}}{\sqrt{S_{xx} \times S_{yy}}}$ $= \frac{3355}{\sqrt{1368 \times 8250}}$ $= 0.99867\ldots =$ **0.999 (3 s.f.)**	**M1** **A1** **2**	Calculator or formula correctly used or equivalent (may be implied) Correct answer, a.r.t. 0.999
(ii) $b = \frac{S_{xy}}{S_{xx}} = \frac{3355}{8250} = \mathbf{0.406666...}$ $a = \bar{y} - b\bar{x} = 80.3 - (0.40666\ldots) \times 55$ $=$ **57.93333** Equation of line $\mathbf{y = 0.407x + 57.9}$	**M1** **A1** **M1** **A1** **4**	S_{xy}/S_{xx} used a.r.t. 0.407 for *b* Using with *their b* $y = 0.407x + 57.9$, c.a.o
(iii) When $x = 45$, $y = 0.407 \times 45 + 57.9 = 76.215$ $=$ **76.2 (3 s.f.)**	**M1** **A1** **2**	Substituting $x = 45$ into *their* eq. a.r.t 76.2

3 (i) $X \sim \text{Geo}(^1/_{12})$	**B1** **B1** **2**	'Geometric' stated $p = {}^1/_{12}$ stated, dependent on Geometric seen first/with it
(ii) (a) $P(X = 3) = {}^1/_{12} \times ({}^{11}/_{12})^2$ $= 0.07002314\ldots$ $= \mathbf{0.07}$ **or** $\mathbf{{}^{121k}/_{1728}}$	**M1** **A1** **2**	(*their p*) × (their *q*)^2 0.07 or 0.070 or ${}^{121k}/_{1728}$
(ii) (b) $P(X > 5) = P(5 \text{ 'failures'}) = ({}^{11}/_{12})^5 = 0.647227\ldots = \mathbf{0.647}$ (3 s.f)	**M1** **A1** **2**	(*their q*)^5 or a wholly correct $1 - P(X \leq 5)$ a.r.t. 0.647
(iii) $E(X) = {}^1/_p = \mathbf{12}$	**B1** **1**	12, c.a.o.
(iv) It seems unlikely that the prob. of a no. 37 bus would remain constant. You would hope that the longer you waited the more likely it would be that the next bus would be a no. 37.	**B1** **1**	A valid comment as to why the model may not be appropriate
4 (i) P(Mike will gain a top grade) = P(he gains it at 1st sitting or at second sitting or at third sitting) $= {}^{70}/_{100} + {}^{30}/_{100} \times {}^{64}/_{100} + {}^{30}/_{100} \times {}^{36}/_{100} \times {}^{58}/_{100} = \mathbf{0.95464}$ **= 0.955 (3 s.f.) AG**	**M1** **M1** **M1** **A1** **4**	For calculation of 3 probs, with at least one having been obtained using a product At least 2 correct terms added All 3 correct terms added 0.955 c.a.o.
(ii) P(Two out of 3 students gain a top grade) $= {}^3C_1 \times 0.95464^2 \times (1 - 0.95464)$ $= \mathbf{0.124018\ldots}$ **= 0.124 (3 s.f.)**	**M1** **M1** **M1** **A1** **4**	Use of p^2q Use of $3p^2q$ Wholly correct method a.r.t. 0.124, allow 0.123

5 (i) No. of different possible arrangements = 6! = **720**	**B1** **1**	720, c.a.o
(ii) Think of *AE* as one block, so no. of arrangements = 5!. But the two letters would also be together if in the block there was *EA*. So no. of arrangements = 5! × 2! = **240**	**M1** **M1** **M1** **A1** **4**	For 5! used Evidence of trying to deal with *EA* and *AE* Wholly correct method 240 c.a.o.
(iii) BY COMBINATIONS P(no vowels) = $\frac{{}^4C_3}{{}^6C_3} = {}^4/_{20} = {}^1/_5 = 0.2$ Therefore P(at least 1 vowel) = 1 – 0.2 = **0.8** ……………………………………….. **OR** P(1 vowel) + P(2 vowels)= $\frac{({}^2C_1 \times {}^4C_2) + ({}^2C_2 \times {}^4C_1)}{{}^6C_3}$ ……………………………………………… **OR BY PROBABILITIES** P(no vowels) = $1 - ({}^4/_6 \times {}^3/_5 \times {}^2/_4)$ **OR** P(1 vowel) or P(2 vowels) = $3({}^2/_6 \times {}^4/_5 \times ¾) + 3({}^2/_6 \times {}^4/_5 \times ¼)$	 **M1** **M1** **M1** **A1** …….. **M1** **M1** **M1** **A1** …….. **M1** **M1** **M1** **A1** **4**	 ${}^6C_3$ seen or implied Subst. correct attempt at finding the number of selections with at least 1 vowel Wholly correct method 0.8 or ${}^{4k}/_{5k}$ ……………………………………… ${}^6C_3$ seen or implied ${}^2C_1$ x ${}^4C_2$ **OR** ${}^2C_2$ x ${}^4C_1$ wholly correct method 0.8 or ${}^{4k}/_{5k}$ ……………………………………… Subst. Attempt at one of P(0 vowels) or P(1 v) or P(2 v), where 3 termed products with at least 2 correct probs. One correct prob. from P(0), P(1), P(2) Wholly correct method 0.8 or ${}^{4k}/_{5k}$

6 (i)

Length, l, in cm	Freq dens	Frequency density × width	Freq
$0 \le l < 3$	3	3 3	**9**
$3 \le l < 6$	5	3 5	**15**
$6 \le l < 10$	4	4 4	**16**
$10 \le l < 15$	1	5 1	**5**
$15 \le l < 25$	0.5	10 0.5	**5**

Mark	Guidance
	ect
M1	Evidence of use of freq. density × width
M1	At least 2 of the missing freqs. correct
A1	All the missing freqs. corr
3	

(ii)

Centre, x	Freq. f	xf	x^2f
1.5	9	13.5	20.25
4.5	15	67.5	303.75
8	16	128	1024
12.5	5	62.5	781.25
20	5	100	2000
TOTAL	50	371.5	4129.25

Working	Mark	Guidance
Mean = $\Sigma(xf)/\Sigma f = {}^{371.5}/_{50}$ = **7.43**	**M1**	Use of $\frac{\Sigma xf}{\Sigma f}$
S.d. = $\sqrt{[\Sigma(x^2f)/\Sigma f - (\text{mean})^2]}$ =	**A1**	7.43 , c.a.o.
$= \sqrt{[{}^{4129.25}/_{50} - 7.43^2]} = 5.232599\ldots$	**M1**	Use of $\frac{\Sigma x^2 f}{\Sigma f}$
= **5.23 (3 s.f.)**	**M1**	$-$ (mean)^2
	M1	Wholly correct method including √
	A1	a.r.t. 5.23
	6	

7 (i) Distribution of J		
<table><tr><td>j</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>$P(J = j)$</td><td>$^1/_8$</td><td>$^3/_8$</td><td>$^3/_8$</td><td>$^1/_8$</td></tr></table>	**M1**	Evidence of use of $^3C_r p^r q^{3-r}$ in either table
	M1	At least one new correct (value, probability) pair in J's table
Distribution of D	**A1**	J's table wholly correct
<table><tr><td>d</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>$P(D = d)$</td><td>$^1/_{27}$</td><td>$^6/_{27}$</td><td>$^{12}/_{27}$</td><td>$^8/_{27}$</td></tr></table>	**M1**	At least one new correct (value, probability) pair in D's table
	A1	D's table wholly correct
	5	
(ii) $P(X = 2)$ $= P(J = 3, D = 1) + P(J = 2, D = 0)$	**M1**	Splitting into cases (3,1) OR (2,0). One of these cases is sufficient.
$= {^1/_8} \times {^6/_{27}} + {^3/_8} \times {^1/_{27}} = {^9/_{216}} =$	**M1**	Two correct cases (two termed products) considered, from their table if necessary
	A1	Both products correct
$\mathbf{^1/_{24}}$ **AG**	**A1**	Wholly correct method
	4	
(iii) P(At least 5 heads) $= P[(J = 2, D = 3) + (J = 3, D = 2) + (J = 3, D = 3)]$	**M1**	Splitting into cases (3,2), (2,3), (3,3), with at least 2 cases correct, based on their table if necessary
$= {^3/_8} \times {^8/_{27}} + {^1/_8} \times {^{12}/_{27}} + {^1/_8} \times {^8/_{27}} = {^{44}/_{216}} =$	**M1**	One correct product (their product for (3,2) or (2,3) or (3,3))
$^{11}/_{54} = 0.203703\ldots =$ **0.204** (3 s.f.)	**M1**	Wholly correct method
	A1	a.r.t. 0.204 or $^{11k}/_{54k}$
	4	